CALIFORNIA INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

ME 96
Piezoelectric Beam Experiment

1. Overview of the experiment

This experiment involves vibration of a piezoelectric beam. Measurements are made to determine properties of the driving circuit, and to determine the fundamental frequency of beam vibration. The student is exposed to measurement techniques, data acquisition, and analysis.

Beam vibration is excited by a strip of piezoelectric material attached to the beam which changes shape when a voltage is applied across it. When the piezoelectric material is in a circuit with an alternating current, the material vibrates at the frequency of the current. By tuning the frequency of the input current, we can change the vibration frequency until it meets the resonant frequency of the beam.

The beam's resonant frequency depends on the mass distribution of the beam (detailed analysis in sections 2 and 3 below).

A laser displacement sensor is used in this experiment to measure beam deflection and find the fundamental frequency. A detailed booklet of instructions for the laser displacement sensor system can be found in the laboratory next to the setup.

2. Analysis of Beam Vibration

Consider a beam of uniform cross section, A, and density, r. At some position x within the beam, a balance of forces and moments can be drawn using $f(x)$ as some distributed load on the beam. The resulting equations for the balance of the force and moments are as follows (assuming that dx approaches zero):

$$-f(x) = \frac{dV}{dx}$$

(1)

and

$$V = \frac{dM}{dx}$$

(2)

These equations can be combined with the moment-deflection equation:

$$M = -EI\frac{\partial^2 v}{\partial x^2}$$

(3)

These equations can be combined to give:

$$EI\frac{\partial^4 v}{\partial x^4} = f(x)$$

(4)

The distributed load is the inertial load due to vibration and can be represented as the product of the mass per unit length and the cross sectional area in the direction opposite of the acceleration:

$$f(x) = -\rho A\frac{\partial^2 v}{\partial t^2}$$

(5)

Combining equations (4) and (5) results in the following fourth order partial differential equation:

$$EI\frac{\partial^4 v}{\partial x^2} + \rho A\frac{\partial^2 v}{\partial t^2} = 0$$

(6)

This differential equation can be solved subject to specified boundary and initial conditions. The above analysis assumes that there is no damping of the vibrating beam, and hence the vibration of the beam would be of constant amplitude. However, in real systems damping is usually always present due to friction. In a vibrating beam the friction is internal to the medium, and is associated with the energy being dissipated randomly in the crystal lattice. The energy dissipation manifests itself as an internal heating of the beam. The frictional damping reduces the amplitude of vibration over time. For a vibrating system with a single degree of freedom (harmonic oscillator), the motion of the system is governed by the following equation:

$$y = a_1 \cos(pt) + a_2 \sin(pt)$$

(7)

where p is the frequency of vibration and is related to the period of vibration by $\tau = 2\pi p$, and a_1 and a_2 are unknown constants. If the system is damped and the damping force is proportional to velocity (as is the case for internal friction), the motion of the damping system is governed by the following equation:

$$y = e^{-nt}(a_1 \sin(qt) + a_2 \cos(qt))$$

(8)

where the n is the damping constant and vibratory motion now has the period:

$$\tau = \frac{2\pi}{q} = \frac{2\pi}{\sqrt{p^2 - n^2}}$$

(9)

Note that if *n* is small relative to *p*, the period of vibration is close to the value obtained without damping.

3. Analysis of Beam Vibration

Using the analysis above we want to determine the fundamental frequency of vibration for the beam used in the experiment (both axes). To find the value, it is useful to follow the following steps.

- Assume that separation of variables can be used to solve equation (6).
- Assume that the beam is undamped. Show that the solution for the vibration of the beam as a function of time takes the form given in equation (7).
- The solution of the differential equation for the *x*–variation is given by:

  $$b_1 \sin(\lambda x) + b_2 \cos(\lambda x) + b_3 \sinh(\lambda x) + b_4 \cosh(\lambda x)$$

  (10)

  where the *b*'s are unknown constants. This solution can also be rewritten as:

  $$c_1(\cos\lambda x + \cosh\lambda x) + c_2(\cos\lambda x - \cosh\lambda x) + c_3(\sin\lambda x + \sinh\lambda x) + c_4(\sin\lambda x - \sinh\lambda x)$$

  (11)

  which is a useful representation for the present configuration.
- Using appropriate boundary conditions for the cantilever beam, find the fundamental frequency in terms of *E*, *I*, *A*, and *ρ*. You will need to know the zeros of the transcendental equation, $\cos(\lambda L)\cosh(\lambda L) = -1$, which are λL = (1.875, 4.694, 7.855, 10.996, …). Note you do not need initial conditions to solve for the frequency.

5. Lab Report

You must address the following points in your report. However, do them in any sensible order- you don't need to stop in the middle of taking data to do a detailed data analysis.

1. Make a sketch in your notebook of the experimental setup, and give a short description of the major components. Include any relevant dimensions you will need for the analysis.
2. Familiarize yourself with the laser displacement sensor, oscilloscope, and data acquisition system (computer). Record the bandwidth, sampling rate, and resolution of the oscilloscope.

3. Use the oscilloscope to observe the output of the inverter circuit (CH 2). First turn on the power supply, and turn the voltage knob to a low voltage (5-10 V). Adjust the oscilloscope so that the signal is visible, and sketch the signal in your lab notebook (be sure to note V/div and sec/div). Using a small screwdriver, tune the potentiometer on the inverter circuit and make a note of its effect on the oscilloscope reading. Find the maximum and minimum frequencies of this inverter circuit.
4. Find the range of voltages for which the circuit amplitude is within the range of the oscilloscope window. In this range, find the relationship between power supply voltage and circuit amplitude (remember to make a note of error values). This relationship will be extrapolated for higher voltages used later in the lab.
5. Using the oscilloscope and the output from the laser displacement sensor, determine the fundamental vibrational frequency of the beam. Make several measurements, so that you can estimate the repeatability. Note that the best accuracy is obtained by measuring times between zero-crossings, not between peaks or valleys.
6. Now measure the fundamental vibrational frequency using the data acquisition system on the computer and the FFT (Fast Fourier Transform) routine. Repeat several times.
7. Examine the effect of sampling rate and total number of samples on the FFT results. Include at least one measurement with a sampling rate less than twice the frequency of oscillation. Make a plot of the measured fundamental frequency vs. sampling rate.
8. Think of a way to *carefully* add small amounts of weight to the tip of the cantilever. Observe with either the oscilloscope or the data acquisition system how this added weight affects the fundamental frequency.

6. Prelab

1. What is a piezoelectric material, and describe 4 examples of how such material is used.
2. If you know the dimensions and weight distribution of the beam, you can calculate the resonant frequency. Determine the fundamental frequency of vibration for a beam (uniform cross-section) in terms of $E, I, \rho,$ and A by solving eq (6). (Hint: start with separation of variables to get X(x) in the form of eq (10) and (11)).
3. Since this is not a simple rectangular beam like the large cantilever experiment, it is not very straightforward to calculate this setup's resonant frequency. What is the resonant frequency reported by the specs? If we add or subtract small amounts of weight from the tip, how will this resonant frequency change?