CALIFORNIA INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

ME 96
Vibrations Experiment

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1 Introduction

In many engineering fields, the input signal to a dynamical system is unknown, but the output signal can be observed and measured. An example of such a system is a seismograph used to measure the strength of an earthquake. Often the characteristics of the input signal can be determined by constructing an analytical model of the dynamical system and examining the response of the system to different input signals. In addition, the dynamic characteristics of various mechanical systems such as an automobile suspension are evaluated by observing the output response to various input test signals.

This experiment is designed to illustrate the dynamic response of a mass-spring-damper system. The input signal is sinusoidal with variable frequency. The movement of two masses is measured with linear variable-displacement transformers (LVDT).

1.1 Description of the experiment

Figure 1 shows a simplified dynamic model of the experiment. The system is composed of two large masses of mass m (approximately 700 grams) on sliding tracks. The masses are separated by linear springs with spring constant k. There is also an air dashpot with a viscous friction coefficient b. The system is driven by a variable speed motor with an eccentric shaft to provide a sinusoidally oscillating input. There are three LVDT to measure the displacement of the mass from the stationary reference frame: one on each of the two masses and one on the driver plate. The output from the sensors is sent through the signal

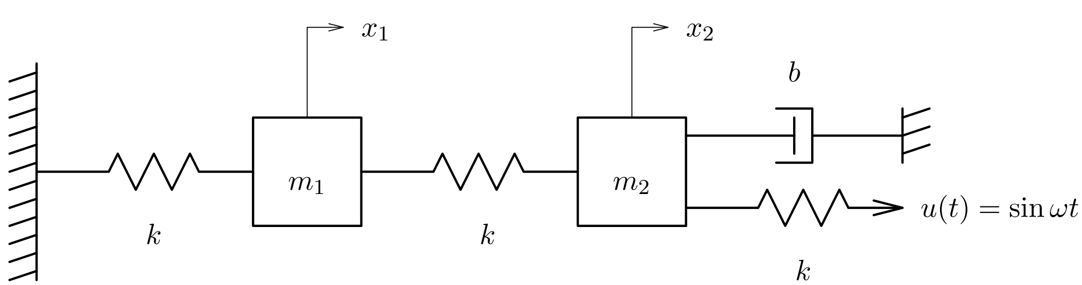

Figure 1: Simplified dynamic model of experiment.

conditioner and into the data acquisition system. The program 'vibrations' can be used to monitor the signals.

1.2 System model

By doing a force balance on each of the masses, two coupled linear ordinary differential equations are determined that we use to model the system. Let $\mathbf{X} = [x_1\ \dot{x}_1\ x_2\ \dot{x}_2]^T$ be the state vector of this system, where x_1 and x_2 are the respective centers of mass. The dynamic model has the form $\dot{\mathbf{X}} = A\mathbf{X} + Bu$, where:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2(k/m) & 0 & (k/m) & 0 \\ 0 & 0 & 0 & 1 \\ (k/m) & 0 & -2(k/m) & -(b/m) \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ (k/m) \end{bmatrix} \tag{1}$$

The transfer function can be derived as: $C(sI - A)^{-1}$ where A and B are as above, and C is the "output matrix." In this case, if one is interested in measuring the response of both x_1 and x_2 then C takes the form:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{2}$$

The characteristic polynomial (e.g., $D(s) = \det(sI - A)$) of this system has the form:

$$D(s) = s^4 + \zeta s^3 + 4\omega_n^2 s^2 + 2\zeta\omega_n^2 s + 3\omega_n^4 \tag{3}$$

where $\zeta = b/m$ and $\omega_n^2 = k/m$. Then the transfer functions from $u(s)$ to $x_1(s)$ and $x_2(s)$ are of the form:

$$\begin{bmatrix} \frac{x_1(s)}{u(s)} \\ \frac{x_2(s)}{u(s)} \end{bmatrix} = \begin{bmatrix} \frac{\omega_n^4}{D(s)} \\ \frac{\omega_n^2\left(s^2+2\omega_n^2\right)}{D(s)} \end{bmatrix}. \tag{4}$$

An alternate approach to finding the transfer function is to use Laplace transforms. The transfer function is defined as the ratio of the Laplace transform of the output function to the Laplace transform of the input or driving function. Using this approach the transfer function is the same as given in Equation (4).

2 Experiments

2.1 Calibration of LVDT transducers

Calibrate the LVDT transducer by determining the displacement versus transducer voltage output curve. The calibration can be done by simply moving the mass a known distance. Check the linearity of the device and be sure to take measurements over the entire range of motion of the masses.

2.2 Response of System to Step Input

Disconnect mass 1. For the single-mass, single-spring system, record the response of the damped and undamped system to a step input. This can be obtained by moving mass 2 with your hand and recording the output. This information will be used in determining k and b.

2.3 Frequency Response (Bode Plot) of the Undamped System

Using the results of Sections 2.1 and 2.2 you should construct plots of the theoretical frequency response. Using these plots as a guide, you should measure the frequency response of the undamped system. The term frequency response refers to the steady-state response to a sinusoidal input. The frequency response information is presented in two separate figures, one giving the magnitude of the ratio of the output to input signals as a function of frequency, and the other presenting the phase angle between the input and output signals versus frequency. The figures are usually presented on a log-log scale for the magnitude and a log-linear scale for the phase. This representation of the frequency response is referred to as a *Bode plot*.

Disconnect and remove the damper from mass 2. To find the frequency response, you want to examine several different input frequencies and record the displacement of the input signal and the displacement of mass 1 and 2. Start with a low frequency and cover a frequency range that includes the first two resonant frequencies. Be sure to take enough data to characterize each of the resonant frequencies.

2.4 Frequency Response of the Damped System

Reconnect the dashpot to mass 2. Set the damper to a relatively low damping coefficient and do not readjust the knob during the course of your experiments. As done in Section 2.3, record the displacement of the input signal and the response of mass 1 and 2.

(Note: be sure to take the response of the damped system with the same damping settings as you had for the step response. The damper has a small knob which controls the amount of damping. If this knob has been moved, you will have to take another step response at this time to compute the correct damping factor.)

3 Analysis of Experimental Model

To match the experimental data to the analytical model, we make a number of simplifying assumptions:

1. Assume that the friction of the LVDT transducer is very small and neglect stiction in the sliding bearings. Thus the friction will be dominated by the air dash pots. This dash pot can nominally be modelled as a viscous friction device, with viscous damping coefficient b.

2. Neglect the mass of the structure connected to the eccentric input shaft.

3. Assume that both large masses have uniform mass, m.

4. Assume that all 3 springs are linear and have uniform spring constant k.

4 Lab Report

1. Sketch and briefly discuss the experimental setup.

2. Record the calibration data, and estimate the uncertainty in the amplitude measurement. Can the transducer be modelled as linear?

3. Measure the step response and the undamped and damped frequency response, as described in the text. Record all data.

4. From your measurements, estimate as accurately as you can the model parameters (spring constants, damping coefficients). There are two ways you might do this. You could determine them from the vibration period and decay rate of the step-response time series data. Alternatively, you could work in the frequency domain, by adjusting the parameters in the theoretical amplitude and/or phase frequency response functions (using the program you wrote) to achieve the best fit to the measured frequency response data. However you do it, make sure your conclusions are self-consistent - i.e., that both the measured step response and frequency response are consistent with the same model parameters.

5. Plot the theoretical phase response vs. frequency for both masses. At what frequency or frequencies do the two masses move in phase? Out of phase? Verify these predictions experimentally.

6. Note that there is a frequency between the two resonances where mass 2 is predicted to remain motionless, even though mass 1 and the input are both moving. Verify this experimentally, and check that the frequency where this occurs is where it is predicted to occur. (This frequency location might be useful in part 4 too)

7. Discuss any problems, discrepancies with theory, possible causes, etc.

8. Time permitting, explore other topics of your own choosing. Can you replace a spring by a nonlinear one, for example? What new phenomena, if any, result from nonlinearity?

5 Advanced Experiment (Optional)

Find or make a nonlinear spring(s). Insert this spring(s) in place of an existing spring, and try to observe the non-linear jump resonance phenomenon which occurs as you slowly increase the frequency of excitation. Alternatively, the springs currently installed in the system are slightly nonlinear. With the proper initial conditions and frequency of excitation, they will exhibit a steady-state nonlinear response. Have the TA show you the conditions which lead to this phenomena.

Prelab

1. Derive Equation 1.
2. Explain what a transfer function is, and derive Equation 4.
3. Write a program or spreadsheet to make Bode plots of the amplitude and phase frequency response from Equation 4.
4. Using the program, explore how damping affects the frequency response functions. Print out and put into your notebook an amplitude Bode plot showing the frequency response for a few damping coefficients ranging from very small to large.