CALIFORNIA INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering

ME 96
Free and Forced Convection Experiment

Revised: March 2003

1 Introduction

The term forced convection refers to heat transport that results from fluid motion caused by external means such as a pump, a fan, or atmospheric winds. The flow velocity depends on the fluid mechanical properties of the system and *not* (ideally) on the heat transfer processes that occur in the system. Typical examples of forced convection include forced-air heating of homes, liquid filled active solar heating systems, and nuclear reactor cooling (during normal operation).

For free or natural convection, the flow velocity depends on *both* the fluid mechanical properties of the system and the heat transfer processes that occur. Free convection is driven by buoyant forces that result from density differences in the convecting fluid. In most situations, the density gradients are caused by temperature variations in the fluid. Examples of free convection include a single-phase closed loop thermosyphone (roughly the condition in a nuclear reactor immediately after pump failure), a Trombe wall (a passive solar heating device), and the shimmering visible above a paved highway on a hot summer day.

The essential difference between free and forced convection manifests itself in the governing equations of the two convection modes. The boundary layer equations for laminar free convection over a heated vertical surface are (neglecting viscous dissipation and pressure gradients):

$$\begin{aligned} \text{Energy:} \quad & u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \\ \text{Momentum:} \quad & u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \end{aligned} \tag{1}$$

where β is the coefficient of thermal expansion, $\beta = -1/\rho(\partial\rho/\partial T)p$. The important point to be noted about these equations is that they are *coupled*. For forced convection the momentum equation does not include the buoyancy term, and hence does not depend on the temperature gradients within the flow.

2 Experimental Apparatus

This experiment is designed to study both free and forced convection heat transfer using a heated plate and a heated cylinder. A schematic of the experimental apparatus is shown in Fig. 1. The plate and the cylinder are shown in Fig. 2. Either surface can be mounted above the fan and the honeycomb flow straightener. The plate can be oriented at different angles to the oncoming flow. The plexiglass side walls are to minimize disturbances from the room.

The heat input is regulated by separate control of three nominally 1 in. $\times$ 5 in. kapton-insulated flexible heaters. The power dissipated through each heater is controlled using a ten-turn potentiometer mounted in the control panel. The power can be calculated from the voltage input as displayed on the control panel, and the heater resistance ($60\Omega \pm 1\%$). The maximum input voltage is 21 volts. The central heater serves as the experimental heater, and the two side heaters act as guard heaters. The power input to the guard heaters should be adjusted so that there is no lateral temperature gradient across the central heater, and hence no heat loss from the central heater to the sides and the metal supports. Typically, the voltage input to the side heaters should be 3 to 7 volts higher than to the central heater. The temperature gradient across the central heater can be monitored using the three aligned thermocouples.

Chromel-alumel thermocouples are attached to the heated surfaces as shown in Fig. 2. There is an additional thermocouple to measure the ambient temperature. The thermocouples are connected to a rotary switch contained in the control panel. The switch is connected to a digital thermometer that contains an electronic reference point. The accuracy of the digital thermometer can be assessed by measuring the temperature of a reference such as an ice bath. In addition, a heat-flux gauge is mounted on one surface of the plate. The calibration for the heat flux gauge is in the lab folder. The output for the heat flux gauge is read on the analogue microvolt meter. In the range of 0–3 mV, the meter has an accuracy of $\pm 5\%$.

The fan is controlled by a DC power supply. The flow velocity is measured using a hot-wire anemometer. The anemometer can be inserted through the collar mounted on the plexiglass wall. Be careful not to break the wire.

3 Experiments

3.1 Free Convection

Begin by orienting the plate in the vertical position, and do not use the fan. Set the power to each of the heaters and monitor the surface temperature until it reaches a steady state. Use power settings close to the maximum value. This warm up process may take about 40 minutes.

After the heater reaches a steady state, record the surface temperatures and the ambient temperature. Record the voltage input to the heaters, and the output from the heat flux sensor. From the measurements of the heat input one can calculate the surface heat flux, q'' in $\mathrm{W/m^2}$ by dividing the input power by the surface area. Remember that the heater has two sides. Check to see if the two methods for determining q'' agree. If the difference is more than 10% and the measurement from the heat flux gauge is lower, reduce the heat input to the central heater.

Without changing the input power, change the plate orientation. Again allow the surface to equilibrate. Record the temperatures and the voltage output from the heat flux gauge. Try at least two different orientations.

Natural convection experiments should be performed in quiescent environments. Room currents will affect the results.

3.2 Forced Convection

Keep the input power at the same value. With the plate in the vertical position, turn on the fan to a low speed. Use the anemometer to record the incoming flow velocity over the central heater. Allow the plate to equilibrate, and record the temperatures and the heat flux. Record the conditions for at least three different fan speeds. The warm-up time should be faster than for the free-convection conditions.

Also try at least one experiment with the plate at a different inclination.

3.3 Flow Over a Cylinder

Disconnect the plate and connect the cylinder. Orient the cylinder so that there is a thermocouple directly at the bottom and at the top. Repeat the measurements for natural convection and for forced convection using several different flow speeds.

There is no heat flux sensor in the cylinder. Start with the same heat input settings used for the forced convection experiment; however, the settings may need to be adjusted.

4 Heat-Transfer Predictions

The Nusselt number, Nu is defined as follows

$$Nu = \frac{h\ell_c}{k} = \frac{q''\ell_c}{(T_w - T_a) \cdot k} \tag{2}$$

where h is the heat transfer coefficient, ℓ_c is a characteristic length, T_w and T_a are the wall and ambient temperatures, and k is the thermal conductivity of the fluid. The Nusselt number can be either a local value, Nu_x where ℓ_c is the distance from the beginning of the plate to the location of interest ($\ell_c = x$) and T_w is the temperature at that location $T_w = T(x)$, or the average value, Nu_a, using the total plate length ($\ell_c = L$) and the average temperature. The average temperature is often difficult to define, and so the midpoint temperature (T_w at $x = L/2$) is often used.

For forced convection, the Nusselt number is presented as a function of Reynolds number, $Re = u_\infty \ell_c/\nu$, where u_∞ is the approach velocity, and ν is the kinematic viscosity. For free convection, the appropriate number is the Rayleigh number, $Ra = g\beta(T_w - T_a)\ell_c^3/\nu\alpha$. The coefficient of thermal expansion for an ideal gas is equal to the inverse of the absolute fluid temperature. All properties should be evaluated at the film temperature, $T_f = (T_a + T_w)/2$.

For laminar forced convection ($Re_L < 5\times10^5$), over a constant heat-flux surface, the theoretical local Nusselt number is

$$Nu_x = 0.453 Re_x^{1/2} Pr^{1/3} \tag{3}$$

and the average value is

$$Nu_a = 0.679 Re_L^{1/2} Pr^{1/3} \tag{4}$$

Pr is the fluid Prandtl number, $Pr = 0.7$ for gases.

For laminar free convection of air over a vertical plate ($0 < Ra_L < 10^9$), the average value is

$$Nu_a = 0.68 + 0.52 Ra_L^{1/4} \tag{5}$$

For flow over a heated cylinder, the characteristic length is the cylinder diameter. The Nusselt number based on the cylinder diameter for air flow is ($10^{-5} < Ra_D < 10^{12}$)

$$Nu_a = \left[0.6 + 0.32 Ra_D^{1/6}\right]^2 \tag{6}$$

For forced convection, the Nusselt number for air flow is ($Re_D Pr > 2$)

$$Nu_a = 0.3 + 0.54 Re_D^{1/2} Pr^{1/3} \left[1 + \left(\frac{Re_D}{28200}\right)^{5/8}\right]^{4/5} \tag{7}$$

Remember that the heated surfaces lose heat by conduction, convection and radiation. The effects of conduction are minimized using the guard heaters. The radiation loss, however, should be accounted for. The radiation loss from a surface can be estimated from the following equation:

$$q'' = \epsilon\sigma(T_w^4 - T_a^4) \tag{8}$$

where ϵ is the surface emissivity, and σ is the Stefan–Boltzmann constant.

5 Lab Report

1. For the natural convection results, plot the average Nusselt number as a function of inclination angle. For each inclination you can obtain two Nusselt numbers corresponding with positive and negative angles from the vertical. Also show on the graph the theoretical value for a vertical plate. Why are there differences?

2. For the forced convection results, plot the average Nusselt number as a function of Reynolds number. Show theoretical values. Also indicate results from the different orientations.

3. For the cylinder, plot the Nusselt number as a function of Reynolds number. Show the theoretical values. Determine the Nusselt number for natural convection and compare it to the theoretical value.

6 Discussion

In the discussion section of the lab report, the following issues should be addressed.

1. How large is the radiation contribution for each of the flows?
2. How well do the theoretical and experimental values compare? What are possible reasons for the differences?
3. How does the angle of attack of the plate affect the heat transfer?

7 References

[1] F. P. Incropera and D. P. DeWitt. *Fundamentals of Heat and Mass Transfer.* John Wiley and Sons, New York, 1985.

[2] J. P. Holman. *Heat Transfer.* McGraw–Hill, Hew York, 1981.

Prelab

It may be helpful to refer to a heat transfer textbook, such as reference [1].

1. What is the Prandtl number?
2. What are typical Prandtl numbers for air, for a liquid metal, and for motor oil?
3. It may be shown that for laminar free convection from a vertical plate, equations (1) admit a *similarity solution*, which is one that combines the two independent variables into only one. For these equations, the solution is as follows. Define the *Grashoff Number* by:

$$Gr_x = \frac{g\beta(T_s - T_\infty)x^3}{\nu^2}$$

and let

$$\eta = \frac{y}{x}(\frac{Gr_x}{4})^{\frac{1}{4}}.$$

Then it may be shown that the vertical velocity u may be computed in terms of a function $f(\eta)$ that depends only on η:

$$u = \frac{2\nu}{x} Gr_x^{\frac{1}{2}} \frac{\partial f}{\partial \eta},$$

and the temperature profile may be determined from a nondimensional temperature:

$$T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

that also only depends on η.

Putting these definitions into equations (1) and simplifying, the equations become:

$$\begin{aligned} \frac{\partial^3 f}{\partial \eta^3} + 3(f)\frac{\partial^2 f}{\partial \eta^2} - 2(\frac{\partial f}{\partial \eta})^2 + T^* &= 0, \\ \frac{\partial^2 T^*}{\partial \eta^2} + 3Pr(f)(\frac{\partial T^*}{\partial \eta})^2 &= 0. \end{aligned}$$

The boundary conditions are:

$$\begin{aligned} f(0) = \frac{\partial f}{\partial \eta}(0) = 0 \quad &; \quad T^*(0) = 1 \\ \frac{\partial f}{\partial \eta}(\infty) = 0 \quad &; \quad T^*(\infty) = 0. \end{aligned}$$

These two coupled ODEs may solved numerically for a specified Prandtl number of the fluid. Write a program or spreadsheet to solve them.

Make a plot of $\frac{\partial f}{\partial \eta}$ vs. η and T^* vs. η for a Prandtl number corresponding to air, and then for Prandtl numbers typical of liquid metal and motor oil.

What can you say about the temperature field in physical coordinates from this solution? Does the trend with Prandtl number make sense?