Optical Ray Tracing

An introduction to the use of lenses to solve optical applications can begin with the elements of ray tracing. Figure 1 demonstrates an elementary ray trace showing the formation of an image, using an ideal thin lens. The object height is $y_1$ at a distance $s_1$ from an ideal thin lens of focal length $f$. The lens produces an image of height $y_2$ at a distance $s_2$ on the far side of the lens.

By ideal thin lens, we mean a lens whose thickness is sufficiently small that it does not contribute to its focal length. In this case, the change in the path of a beam going through the lens can be considered to be instantaneous at the center of the lens, as shown in the figure. In the applications described here, we will assume that we are working with ideally thin lenses. This should be sufficient for an introductory discussion. Consideration of aberrations and thick-lens effects will not be included here.

Three rays are shown in Figure 1. Any two of these three rays fully determine the size and position of the image. One ray emanates from the object parallel to the optical axis of the lens. The lens refracts this beam through the optical axis at a distance $f$ on the far side of the lens. A second ray passes through the optical axis at a distance $f$ in front of the lens. This ray is then refracted into a path parallel to the optical axis on the far side of the lens. The third ray passes through the center of the lens. Since the surfaces of the lens are normal to the optical axis and the lens is very thin, the deflection of this ray is negligible as it passes through the lens.

In addition to the assumption of an ideally thin lens, we also work in the paraxial approximation. That is, angles are small and we can substitute $\theta$ in place of $\sin \theta$.

Magnification

We can use basic geometry to look at the magnification of a lens. In Figure 2, we have the same ray tracing figure with some particular line segments highlighted. The ray through the center of the lens and the optical axis intersect at an angle $\phi$. Recall that the opposite angles of two intersecting lines are equal. Therefore, we have two similar triangles. Taking the ratios of the sides, we have

$$\phi = y_1/s_1 = y_2/s_2$$

This can then be rearranged to give

$$y_2/y_1 = s_2/s_1 = M.$$  

The quantity $M$ is the magnification of the object by the lens. The magnification is the ratio of the image size to the object size, and it is also the ratio of the image distance to the object distance.

Gaussian Lens Equation

Let’s now go back to our ray tracing diagram and look at one more set of line segments. In Figure 3, we look at the optical axis and the ray through the front focus. Again looking at similar triangles sharing a common vertex and, now, angle $\eta$, we have

$$y_2/f = y_1/(s_1-f).$$

Rearranging and using our definition of magnification, we find

$$y_2/y_1 = s_2/s_1 = f/(s_1-f).$$

This puts a fundamental limitation on the geometry of an optics system. If an optical system of a given size is to produce a particular magnification, then there is only one lens position that will satisfy that requirement. On the other hand, a big advantage is that one does not need to make a direct measurement of the object and image sizes to know the magnification; it is determined by the geometry of the imaging system itself.

Optical Invariant

Now we are ready to look at what happens to an arbitrary ray that passes through the optical system. Figure 4 shows such a ray. In this figure, we have chosen the maximal ray, that is, the ray that makes the maximal angle with the optical axis as it leaves the object, passing through the lens at its maximum clear aperture. This choice makes it easier, of course, to visualize what is happening in the system, but this maximal ray is also the one that is of most importance in designing an application. While the figure is drawn in this fashion, the choice is completely arbitrary and the development shown here is true regardless of which ray is actually chosen.
Optics

TECHNICAL REFERENCE AND FUNDAMENTAL APPLICATIONS

Application 1: Focusing a Collimated Laser Beam

As a first example, we look at a common application, the focusing of a laser beam to a small spot. The situation is shown in Figure 5. Here we have a laser beam, with radius \( y_1 \) and divergence \( \theta_1 \), that is focused by a lens of focal length \( f \). From the figure, we have \( \theta_2 = y_2 / f \). The optical invariant then tells us that we must have \( y_2 = \theta_1 f \), because the product of radius and divergence angle must be constant.

Application 2: Collimating Light from a Point Source

Another common application is the collimation of light from a very small source, as shown in Figure 6. The problem is often stated in terms of collimating the output from a “point source.” Unfortunately, nothing is ever a true point source and the size of the source must be included in any calculation. In Figure 6, the point source has a radius of \( y_1 \) and has a maximum ray of angle \( \theta_1 \). If we collimate the output from this source using a lens with focal length \( f \), then the result will be a beam with a radius \( y_2 = \theta_1 f \) and divergence angle \( \theta_2 = y_2 / f \). Note that, no matter what lens is used, the beam radius and beam divergence have a reciprocal relation. For example, to improve the collimation by a factor of two, you need to increase the beam diameter by a factor of two.

Application 3: Expanding a Laser Beam

It is often desirable to expand a laser beam. At least two lenses are necessary to accomplish this. In Figure 7, a laser beam of radius \( y_1 \) and divergence \( \theta_1 \) is expanded by a negative lens with focal length \(-f_1\). From Applications 1.1 and 1.2 we know \( \theta_2 = y_2 / f_1 \), and the optical invariant tells us that the radius of the virtual image formed by this lens is \( y_2 = \theta_1 f_1 \). This image is at the focal point of the second lens, \( s_2 = f_2 \), because a well-collimated laser yields \( s_1 = \infty \), so from the Gaussian lens equation \( s_2 = f_2 \). Adding a second lens with a positive focal length \( f_2 \) and separating the two lenses by the sum of the two focal lengths \(-f_1 + f_2\), results in a beam with a radius \( y_3 = y_2 f_2 / f_1 \) and divergence angle \( \theta_3 = y_3 / f_2 \).
The expansion ratio
\[ \frac{y_3}{y_1} = \frac{y_2 f_2}{y_1} = \frac{f_2}{-f_1}, \]
or the ratio of the focal lengths of the lenses. The expanded beam diameter
\[ 2y_3 = 2B_1 f_2 = 2y_1 \frac{f_2}{-f_1}. \]
The divergence angle of the resulting expanded beam
\[ \theta_3 = \frac{y_2}{f_2} = \frac{y_1}{-f_1} \frac{f_2}{f_1} \]
is reduced from the original divergence by a factor that is equal to the ratio of the focal lengths \(-f_1/f_2\). So, to expand a laser beam by a factor of five we would select two lenses whose focal lengths differ by a factor of five, and the divergence angle of the expanded beam would be 1/5th the original divergence angle.

As an example, consider a Newport R-31005 HeNe laser with beam diameter 0.63 mm and a divergence of 1.3 mrad. Note that these are beam diameter and full divergence, so in the notation of our Table 3.1 the beam diameter will be 0.315 mm and \(\theta_1 = 0.65\) mrad.

Extended Source to a Multimode Fiber

The radius of the fiber core will be our \(y_2\).

Small Spot

Application 4: Focusing an Extended Source to a Small Spot

This application is one that will be approached as an imaging problem as opposed to the focusing and collimation problems of the previous applications. An example might be a situation where a fluorescing sample must be imaged with a CCD camera. The geometry of the application is shown in Figure 8. An extended source with a radius of \(y_1\) is located at a distance \(s_1\) from a lens of focal length \(f\). The figure shows a ray incident upon the lens at a radius of \(R\). We can take this radius \(R\) to be the maximal allowed ray, or clear aperture, of the lens.

\[ \frac{2y_3}{y_1} = \frac{2y_1 f_2}{-f_1} = \frac{2(0.315 mm)(250 mm)}{-25 mm} = 6.3 \text{ mm} \]

\[ \theta_3 = \frac{y_1}{-f_1} \frac{f_2}{f_1} = (0.65 \text{ mrad}) \frac{25 mm}{250 mm} \]
\[ = 0.065 \text{ mrad}. \]

For minimal aberrations, it is best to use a plano-concave lens for the negative lens and a plano-convex lens for the positive lens with the plano surfaces facing each other. To further reduce aberrations, only the central portion of the lens should be illuminated, so choosing oversized lenses is often a good idea. This style of beam expander is called Galilean. Two positive lenses can also be used in a Keplerian beam expander design, but this configuration is longer than the Galilean design.

Fiber Optic Coupling

Application 5: Coupling Laser Light into a Multimode Fiber

When we look at coupling light from a well-collimated laser beam into a multimode optical fiber, we return to the situation that was illustrated in Figure 5. The radius of the fiber core will be our \(y_2\). We will have to make sure that the lens focuses to a spot size less than this parameter. An even more important restriction is that the angle from the lens to the fiber \(\theta_2\) must be less than the NA of the optical fiber.

Let’s consider coupling the light from a Newport R-30990 HeNe laser into an F-MSD fiber. The laser has a beam diameter of 0.81 mm and divergence 0.0 mrad. The fiber has a core diameter of 50 \(\mu\)m and an NA of 0.20. Let’s look at the coupling from the beam into the fiber when a Newport M-20X objective lens is used in an F-915 or F-915T fiber coupler.

The objective lens has an effective focal length of 9 mm. In this case, the focused beam will have a diameter of 9 \(\mu\)m and a maximal ray of angle 0.05, so both the...